Quark fragmentation into spin-triplet *S*-wave quarkonium

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Outline

- Leading-power approximation and NRQCD
- Motivation
- Result and Conclusion

Leading-power approximation and NRQCD

Nonrelativistic QCD (NRQCD) factorization

- NRQCD factorization formula for a quarkonium H production is given by

short-distance coefficient (SDC, perturbative, α_s)

$$\sigma[H] = \sum_n \boxed{\hat{\sigma}_{Q\bar{Q}(n)}(\mu_\Lambda)}\boxed{\langle 0|\mathcal{O}^H_{Q\bar{Q}(n)}|0\rangle}$$

long-distance matrix element (LDME, nonperturbative, v)

- ${}^3S_1^{[8]}$, ${}^3P_J^{[8]}$, ${}^1S_0^{[8]}$, and ${}^3S_1^{[1]}$ channels contribute to J/ψ production through order v^4

Leading-power (LP) factorization

- The leading contribution in $1/p_T^2$ can be factorized into a product of parton production cross sections and fragmentation functions:

$$d\sigma_{A+B\to H+X} = \sum_i \int_0^1 dz\, d\hat{\sigma}_{A+B\to i+X}\left(k^+, \mu_f\right) \times D_{i\to H}\left(z=\frac{p^+}{k^+}, \mu_f\right) \equiv \sum_i d\hat{\sigma}_{A+B\to i+X} \otimes \boxed{D_{i\to H}}$$

nonperturbative

where,

$d\hat{\sigma}_{A+B\to i+X}$: single parton i production cross section

$D_{i\to H}$: single parton fragmentation function

k^+: light-cone momentum of parent parton i

p^+: light-cone momentum of parent parton i

μ_f: factorization scale

Leading-power (LP) factorization and NRQCD

- If we apply LP factorization to NRQCD factorization, then we get

$$d\sigma_{A+B\to H+X} = \sum_{n,i} d\sigma_{A+B\to i+X} \otimes \boxed{D_{i\to Q\bar{Q}(n)}} \langle \mathcal{O}^H(n) \rangle$$

perturbative

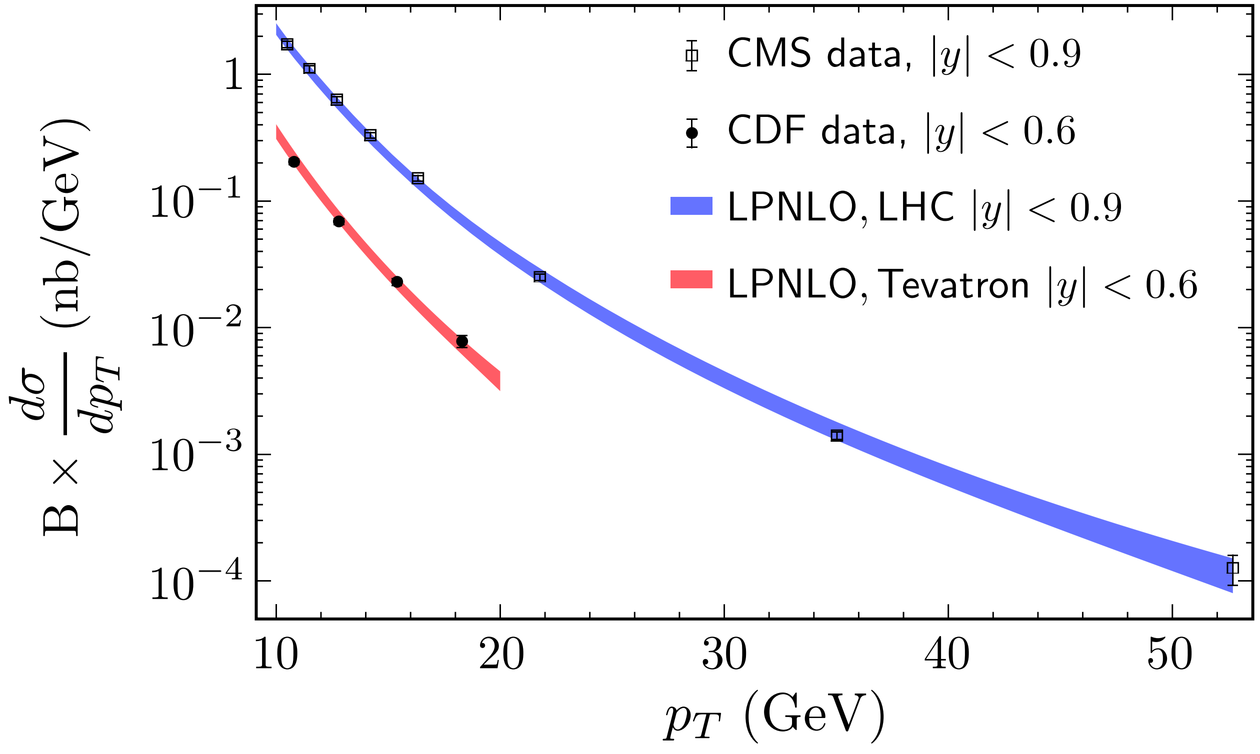
  - At large p_T, the LP contribution becomes dominant Therefore, we can estimate the large p_T behavior by making use of LP factorization.

- By evaluating LP contributions, we estimated the NNLO contributions in α_s to the J/ψ production at the LHC and the Tevatron, which gives the dramatic change in the dominant channel to J/ψ production. (Prof. Jungil Lee's talk)

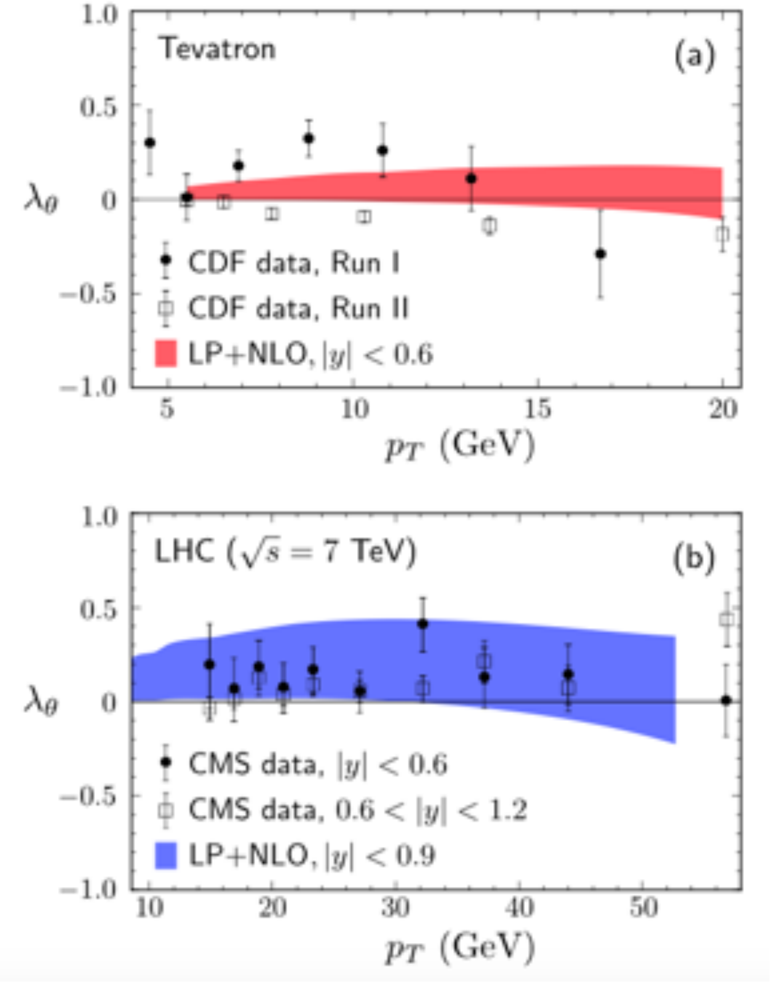
Motivation

Fragmentation contribution to J/ψ production

- Applying the LP factorization to NRQCD enables us to estimate the amount of NNLO contribution to J/ψ production in α_s at large p_T



PRL 113, 022001 (2014)



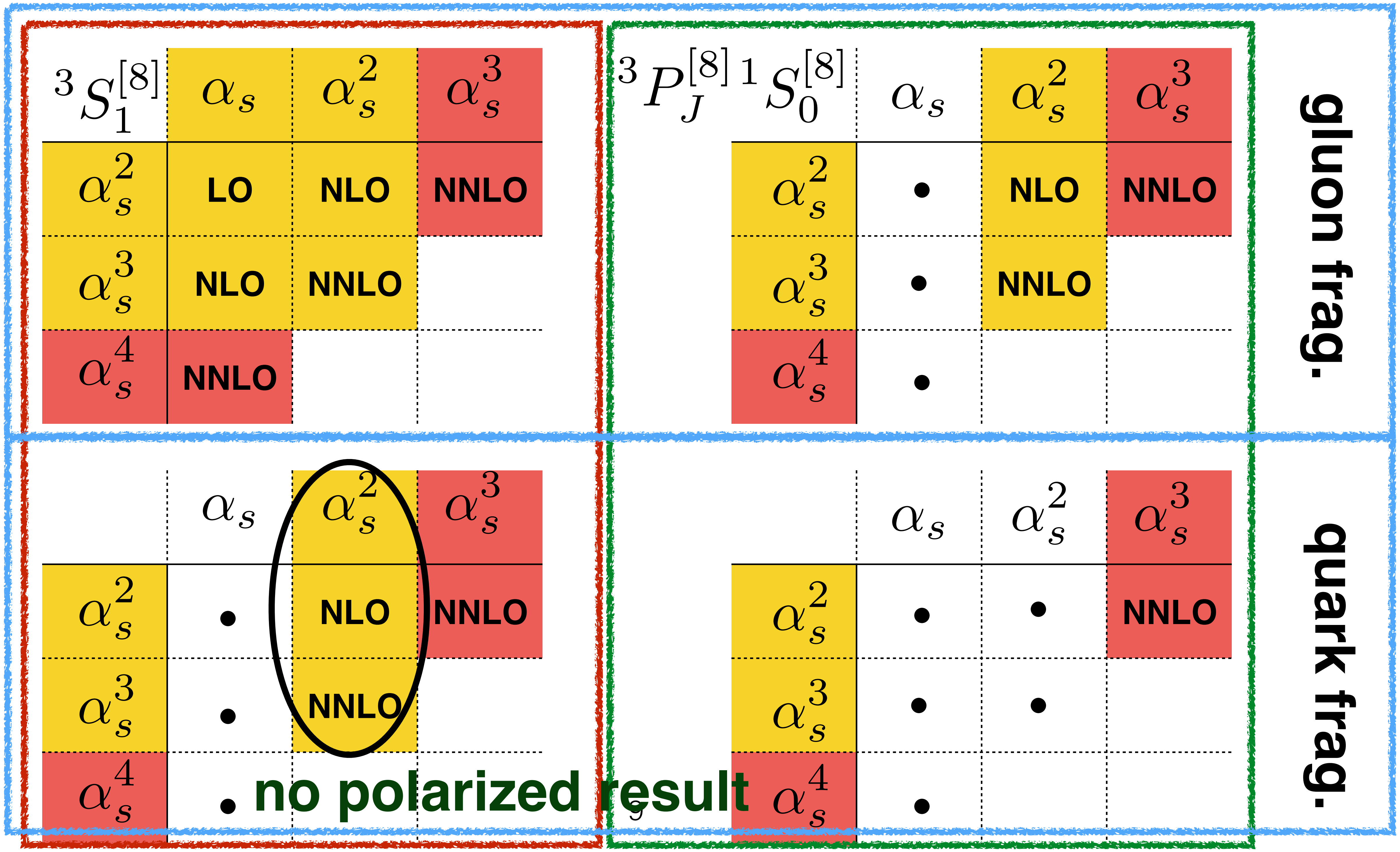
- Details were presented in Prof. Jungil Lee's talk

Missing parts in LP approximation

- The following red-colored parts are missing

Fragmenation function —>

<—Parton production cross section



gluon frag.

$^3S_1^{[8]}$	α_s	α_s^2	α_s^3
α_s^2	**LO**	**NLO**	**NNLO**
α_s^3	**NLO**	**NNLO**	
α_s^4	**NNLO**		

$^3P_J^{[8]}\ ^1S_0^{[8]}$	α_s	α_s^2	α_s^3
α_s^2	•	**NLO**	**NNLO**
α_s^3	•	**NNLO**	
α_s^4	•		

quark frag.

	α_s	α_s^2	α_s^3
α_s^2	•	**NLO**	**NNLO**
α_s^3	•	**NNLO**	
α_s^4	•		

	α_s	α_s^2	α_s^3
α_s^2	•	•	**NNLO**
α_s^3	•	•	
α_s^4	•		

no polarized result

Result

Collins-Soper definition of the quark fragmentation function

$$D_{q\to H}(z) = \frac{z^{d-3}}{N_c \times 4 \times 2\pi}\int_{-\infty}^{+\infty} dx^- e^{-iP^+x^-/z}\mathrm{tr}\left[n\langle 0|\Psi(0)\mathcal{E}^\dagger(0)\mathcal{P}_{H(P,\lambda)}\mathcal{E}(x^-)\bar{\Psi}(x)|0\rangle\right]$$

Initial quark field $\Psi(x)$

gauge link $\mathcal{E}(x^-) = \mathcal{P}\exp\left[+ig_s\int_{x^-}^{\infty} dz^-\, A^+(0^+, z^-, \mathbf{0}_\perp)\right]$

Projector $\mathcal{P}_{H(P,\lambda)} = \sum_X |H(P,\lambda) + X\rangle\langle H(P,\lambda) + X|$

- Following the Collins-Soper definition, we evaluated fragmentation functions in the Feynman gauge

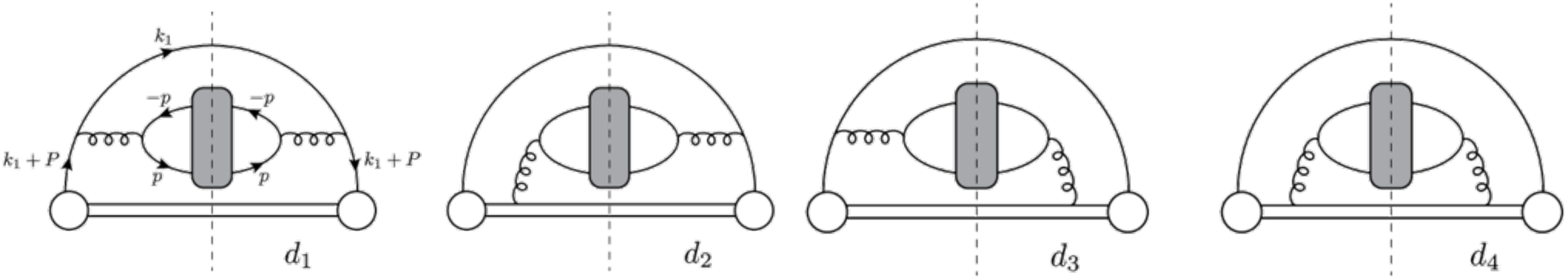
What we consider

1. **Color-octet, different-flavor case:**
   the fragmenting quark is not the same with the constituent of the quarkonium

2. **Color-singlet**

3. **Color-octet, same-flavor case**
   the fragmenting quark is the same with the constituent of the quarkonium

Color-octet, different-flavor case



$$\sum_{\lambda} D^{\overline{\mathrm{MS}}}_{q\to H(\lambda)}(z,\mu) = \frac{\alpha_s^2 C_F}{2m_Q^3}\left\{\frac{z^2-2z+2}{z}\left[\log\frac{\mu^2}{4m_Q^2} - \log(1-z+rz^2)\right]\right.$$
$$\left. -z - \frac{z(1-z)(1+2r)}{1-z+rz^2}\right\}\frac{\langle O^H({}^3S_1^{[8]})\rangle}{3(N_c^2-1)} \qquad \left(r = \frac{m_q^2}{M^2}\right)$$

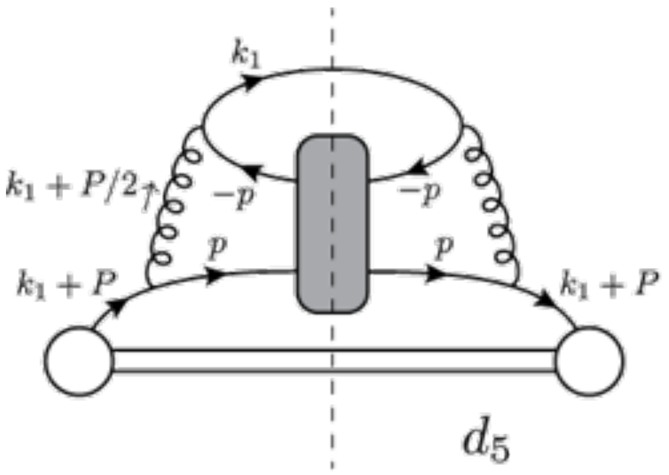
confirms the previous results in PRD53, 1185 (1996) and PRD89, 094029 (2014)

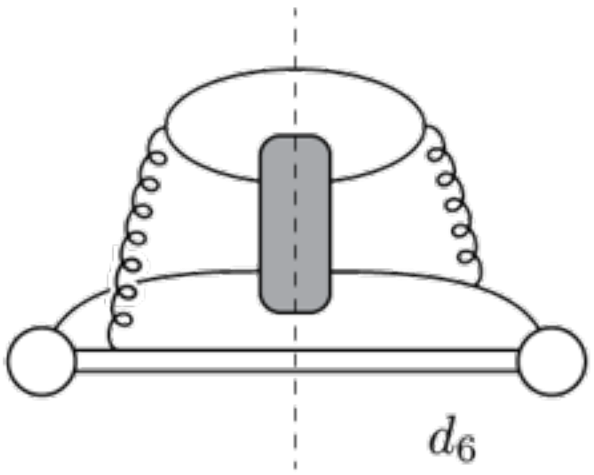
$$D_{q\to H(\lambda=0)}(z) = \frac{\alpha_s^2 C_F}{2m_Q^3}\frac{2(1-z)}{z}\frac{1-z}{1-z+rz^2}\frac{\langle O^H({}^3S_1^{[8]})\rangle}{3(N_c^2-1)}$$

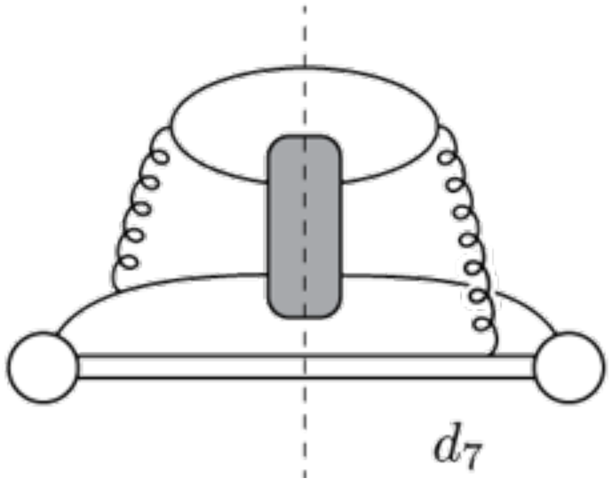
disagrees with the result of H. Zhang in his thesis
He confirms that our result is correct

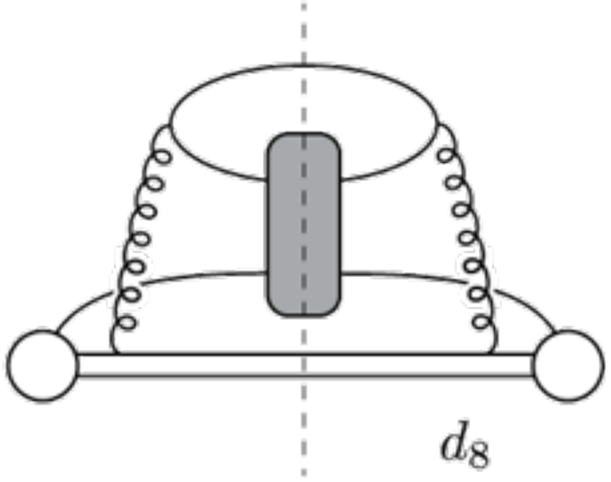
NEW

Color-singlet case









$$\sum_{\lambda=0,\pm 1} D_{Q\to H(\lambda)} = \frac{\alpha_s^2 C_F^2 z(1-z)^2(5z^4-32z^3+72z^2-32z+16)}{9N_c(2-z)^6 m_Q^3}\langle O^H({}^3S_1^{[1]})\rangle$$

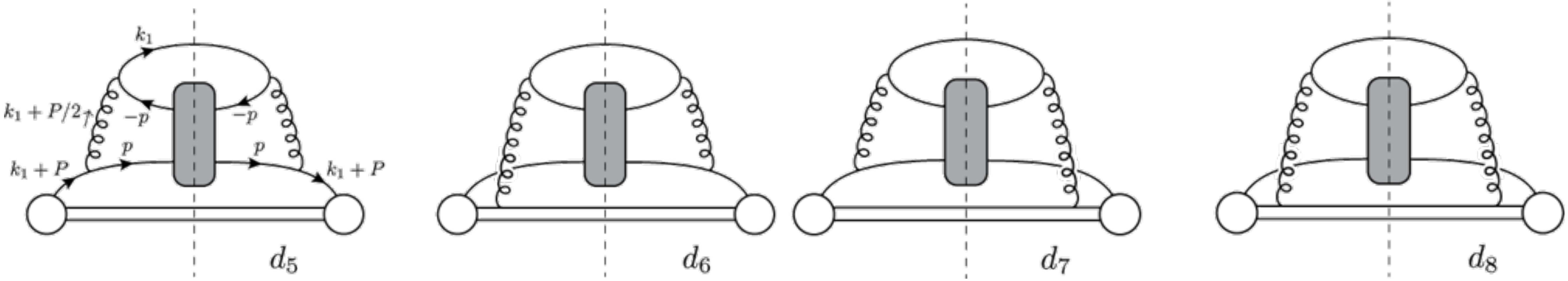
confirms the previous results in PRD48, 4230 (1993) and PRD89, 094029 (2014)

$$D_{Q\to H(\lambda=0)} = \frac{\alpha_s^2 C_F^2 z(1-z)^2(3z^4-24z^3+64z^2-32z+16)}{27N_c(2-z)^6 m_Q^3}\langle O^H({}^3S_1^{[1]})\rangle$$

agrees with the result of H. Zhang in his thesis

NEW

Color-octet, same flavor case (previous result)

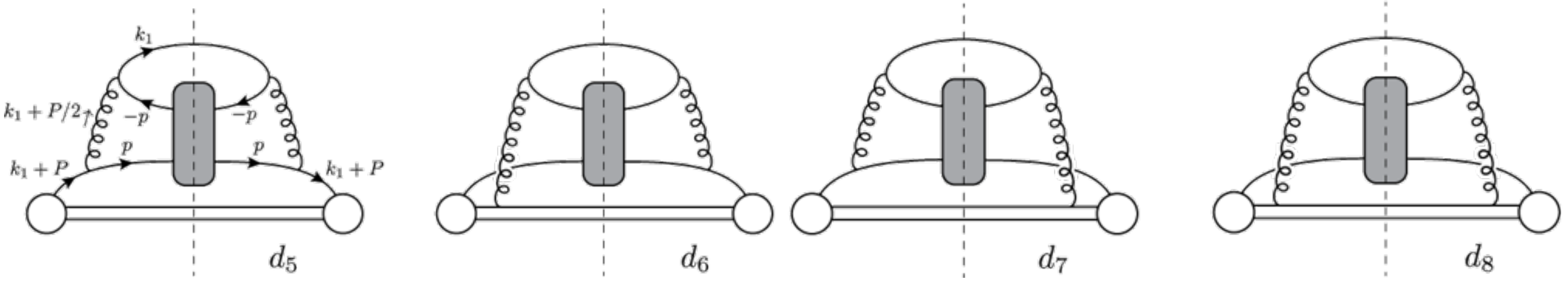


In PRD50, 5664 (1994),

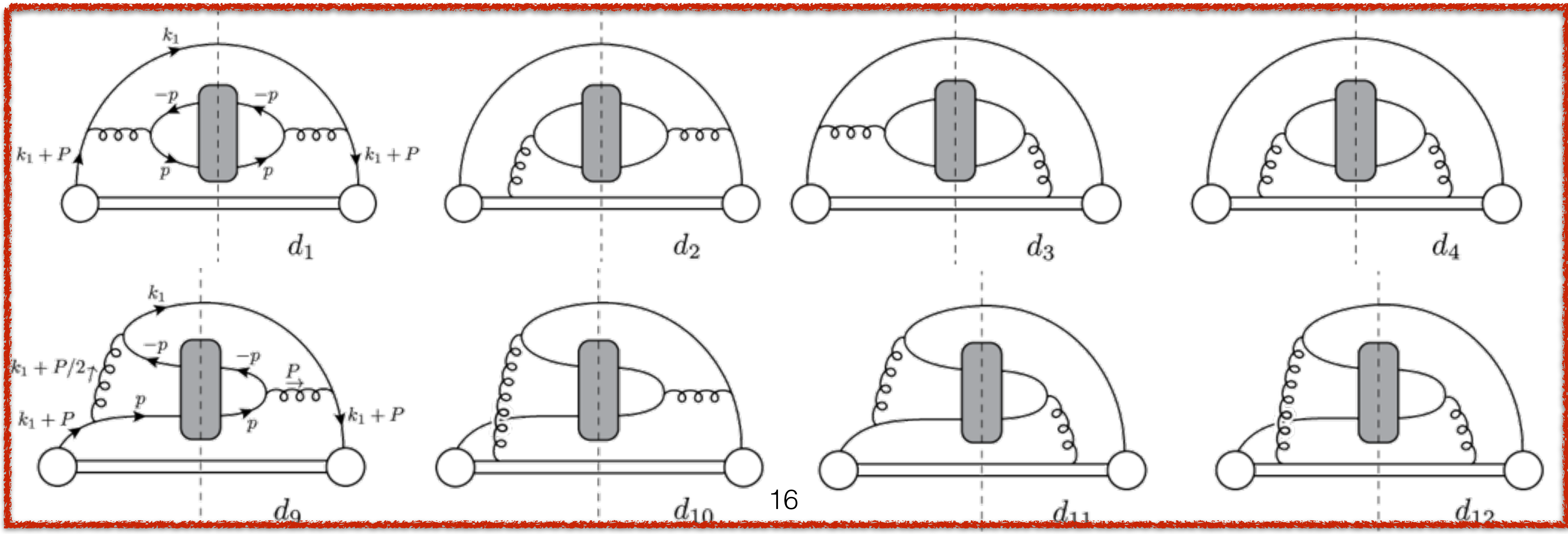
$$D^{[8]}_{c\to c\bar{c}(^3S_1)}(z) = \frac{\alpha_s^2(2m_c)}{162}\frac{z(1-z)^2}{(2-z)^6}\left(16-32z+72z^2-32z^3+5z^4\right)$$

The author evaluated the result from the color-singlet result by changing the color factor only

Color-octet, same flavor case



The author neglected the following diagrams:



Color-octet, same flavor case

$$\begin{aligned}\sum_{\lambda} D^{\overline{\rm MS}}_{Q\to H(\lambda)}(z,\mu) = \frac{\alpha_s^2 C_F}{2zN_c^2(2-z)^6 m_Q^3}\Big[&N_c^2(z^2-2z+2)(2-z)^6\log\frac{\mu^2}{(2-z)^2 m_Q^2}\\ &-N_c^2 z^2(2-z)^4(z^2-10z+10)\\ &+16N_c z(2-z)^2(1-z)(z^3-6z^2+6z-2)\\ &+2z^2(1-z)^2(5z^4-32z^3+72z^2-32z+16)\Big]\frac{\langle O^H(^3S_1^{[8]})\rangle}{3(N_c^2-1)},\end{aligned}$$

confirms the previous result in PRD89, 094029 (2014)

$$\begin{aligned}D_{Q\to H(\lambda=0)}(z) = \frac{\alpha_s^2 C_F(1-z)^2}{3N_c^2 z(2-z)^6 m_Q^3}\Big[&12N_c^2(2-z)^4+12N_c z^2(2-z)^2(4-z)\\ &+z^2(3z^4-24z^3+64z^2-32z+16)\Big]\frac{\langle O^H(^3S_1^{[8]})\rangle}{3(N_c^2-1)}.\end{aligned}$$

agrees with the result of H. Zhang in his thesis

NEW

Conclusion

- We evaluated spin-triplet *S*-wave quark fragmentation functions for the cases in which the color-octet different flavor, the color-octet same flavor, and the color-singlet and their polarized results.

- The new results are useful to analyze the prompt J/ψ production and polarization.

Thank you

Backups

Dokshitzer–Gribov–Lipatov–Altarelli–Parisi (DGLAP) equation

- Leading-log (LL) terms can be summed over all orders in α_s by solving DGLAP equation.

- LO DGLAP evolution equation is given by

$$\frac{d}{d\log\mu_f^2}\begin{pmatrix} D_S \\ D_g \end{pmatrix}(z,\mu_f) = \int \frac{dx}{x}\frac{\alpha_s(\mu_f)}{2\pi}\begin{pmatrix} P_{qq} & 2n_f P_{gq} \\ P_{qg} & P_{gg} \end{pmatrix}\left(\frac{z}{x}\right) \times \begin{pmatrix} D_S \\ D_g \end{pmatrix}(x,\mu_f)$$

where,

P_{ij}: splitting function

$$D_S = \sum_f (D_{q_f \to Q\bar{Q}(n)} + D_{\bar{q}_f \to Q\bar{Q}(n)})$$

$$D_g = D_{g \to Q\bar{Q}(n)}$$

n_f: active quark flavor

Polarization parameter

Polarization parameters

- In experiment, by observing the angular distribution of the lepton pair that are decayed from J/ψ meson, we determine the polarization of J/ψ:

$$\frac{d\Gamma(J/\psi \to l^+ l^-)}{d\cos\theta\, d\phi} \propto 1 + \lambda_\theta \cos^2\theta + \lambda_\phi \sin^2\theta \cos 2\phi + \lambda_{\theta\phi} \sin 2\theta \cos\phi$$

- Here, θ is the polar angle and ϕ is the azimuthal angle of the lepton l^+ in the J/ψ rest frame.

$\lambda_\theta = 1$: Transversely polarized
$\lambda_\theta = 0$: Unpolarized
$\lambda_\theta = -1$: Longitudinally polarized.

Polarization parameters

- In theory, the polarization parameter λ_θ is determined by

$$\lambda_\theta = \frac{\sigma_T - 2\sigma_L}{\sigma_T + 2\sigma_L}$$

- Here, σ_T is the transverse components of the cross section and σ_L is the longitudinal components of the cross section respectively.

Polarization parameter λ_θ

- ${}^3S_1^{[8]}$ and ${}^3P_J^{[8]}$ contributions become 100% transversely polarized at high p_T

  → assumed that ${}^3S_1^{[8]}$ and ${}^3P_J^{[8]}$ are 100 % transverse.

- ${}^1S_0^{[8]}$ contribution is unpolarized.

  → 1/3 Longitudinal, 2/3 transverse.

$$\sigma_T = \frac{d\sigma\left[{}^3S_1^{[8]}\right]}{dp_T} + \frac{\mathrm{d}\sigma\left[{}^3P_J^{[8]}\right]}{dp_T} + \frac{2}{3}\frac{d\sigma\left[{}^1S_0^{[8]}\right]}{dp_T}$$

$$\sigma_L = \frac{1}{3}\frac{d\sigma\left[{}^1S_0^{[8]}\right]}{dp_T}$$

$$\lambda_\theta = \frac{\sigma_T - 2\sigma_L}{\sigma_T + 2\sigma_L}$$

NRQCD LDMEs

NRQCD LDMEs for J/ψ

- v^0: $\langle 0|\mathcal{O}_0^H({}^3S_1^{[1]})|0\rangle = \langle 0|\chi^\dagger\sigma^i\psi\,\mathcal{P}_H\,\psi^\dagger\sigma^i\chi|0\rangle$
- v^2: $\langle 0|\mathcal{O}_2^H({}^3S_1^{[1]})|0\rangle = \frac{1}{2}\langle 0|\chi^\dagger\sigma^i(-\tfrac{i}{2}\overleftrightarrow{\boldsymbol{D}})^2\psi\,\mathcal{P}_H\,\psi^\dagger\sigma^i\chi + \text{H. c.}|0\rangle$
- v^3: $\langle 0|\mathcal{O}_0^H({}^1S_0^{[8]})|0\rangle = \langle 0|\chi^\dagger T^a\psi\,\mathcal{P}_H\,\psi^\dagger T^a\chi|0\rangle$
- v^4(color octet):

$$\langle 0|\mathcal{O}_0^H({}^3S_1^{[8]})|0\rangle = \langle 0|\chi^\dagger\sigma^i T^a\psi\,\mathcal{P}_H\,\psi^\dagger\sigma^i T^a\chi|0\rangle$$

$$\langle 0|\mathcal{O}_0^H({}^3P_0^{[1]})|0\rangle = \frac{1}{d-1}\langle 0|\chi^\dagger(-\tfrac{i}{2}\overleftrightarrow{\boldsymbol{D}}\cdot\boldsymbol{\sigma})\psi\,\mathcal{P}_H\,\psi^\dagger(-\tfrac{i}{2}\overleftrightarrow{\boldsymbol{D}}\cdot\boldsymbol{\sigma})\chi|0\rangle$$

$$\langle 0|\mathcal{O}_0^H({}^3P_1^{[1]})|0\rangle = \langle 0|\chi^\dagger(-\tfrac{i}{2}\overleftrightarrow{D}^{[i,}\sigma^{j]})\psi\,\mathcal{P}_H\,\psi^\dagger(-\tfrac{i}{2}\overleftrightarrow{D}^{[i,}\sigma^{j]})\chi|0\rangle$$

$$\langle 0|\mathcal{O}_0^H({}^3P_2^{[1]})|0\rangle = \langle 0|\chi^\dagger(-\tfrac{i}{2}\overleftrightarrow{D}^{(i}\sigma^{j)})\psi\,\mathcal{P}_H\,\psi^\dagger(-\tfrac{i}{2}\overleftrightarrow{D}^{(i}\sigma^{j)})\chi|0\rangle$$

$$\langle 0|\mathcal{O}_0^H({}^3P^{[8]})|0\rangle = \sum_{J=0,1,2}\langle 0|\mathcal{O}_0^H({}^3P_J^{[8]})|0\rangle$$

NRQCD LDMEs for J/ψ

- Order-v^4(color singlet)

$$\langle 0|\mathcal{O}^H_{4,1}({}^3S_1^{[1]})|0\rangle = \langle 0|\chi^\dagger\sigma^i(-\tfrac{i}{2}\overleftrightarrow{\boldsymbol{D}})^2\psi\,\mathcal{P}_H\,\psi^\dagger\sigma^i(-\tfrac{i}{2}\overleftrightarrow{\boldsymbol{D}})^2\chi|0\rangle$$

$$\langle 0|\mathcal{O}^H_{4,2}({}^3S_1^{[1]})|0\rangle = \frac{1}{2}\langle 0|\chi^\dagger\sigma^i(-\tfrac{i}{2}\overleftrightarrow{\boldsymbol{D}})^4\psi\,\mathcal{P}_H\,\psi^\dagger\sigma^i\chi + \text{H. c.}|0\rangle$$

$$\begin{aligned}\langle 0|\mathcal{O}^H_{4,3}({}^3S_1^{[1]})|0\rangle = \frac{1}{2}\langle 0|&\chi^\dagger\sigma^i\psi\,\mathcal{P}_H\,\psi^\dagger\sigma^i(\overleftrightarrow{\boldsymbol{D}}\cdot g\boldsymbol{E} + g\boldsymbol{E}\cdot\overleftrightarrow{\boldsymbol{D}})\chi\\ &-\chi^\dagger\sigma^i(\overleftrightarrow{\boldsymbol{D}}\cdot g\boldsymbol{E} + g\boldsymbol{E}\cdot\overleftrightarrow{\boldsymbol{D}})\psi\,\mathcal{P}_H\,\psi^\dagger\sigma^i\chi|0\rangle\end{aligned}$$

- Equation of motion eliminates $\langle 0|\mathcal{O}^H_{4,3}({}^3S_1^{[1]})|0\rangle$
- $\langle 0|\mathcal{O}^H_{4,1}({}^3S_1^{[1]})|0\rangle = \langle 0|\mathcal{O}^H_{4,2}({}^3S_1^{[1]})|0\rangle + \mathcal{O}(v^2)$

Charmonium spectroscopy